Engineering Notes

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Sensitivity to Cross-Axis Oscillations in a Single-Axis Nuclear Gyroscope

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Nomenclature

B = applied magnetic field
H = sample angular momentum
^3He = helium^{-3}
M = sample magnetization
γ = gyromagnetic ratio
ω = angular rate

Introduction

NUCLEAR gyro developments are aimed at providing a low-cost, high-reliability alternative to conventional mechanical gyroscopes.[1] By removing the need for precision moving parts, such devices should be inherently insensitive to mechanical shock and vibration; thus, well suited to strapdown systems. Both Litton Guidance and Control[1,2] and Singer-Kearfott[3,4] have developed engineering models of two species nuclear magnetic resonance gyros for strapdown applications. These are both room-temperature devices which utilize continuous optical pumping to sustain operation. Stanford University[5,6] is developing a device which uses a single isotope of helium; and because of the long relaxation times obtainable with ^3He at 4 K, no optical pumping is required to maintain operation.[5]

Although these devices are not subject to damage from shock and vibration, digital simulation studies of the Stanford device have shown that vibrations which cause certain oscillations about cross axes can be troublesome. In particular, if these oscillations occur at the precession frequency of the ^3He, periodic loss of gyro output signal results. This effect, as presented, is directly applicable to the Stanford device only, but would produce undesirable effects in the other nuclear gyro mechanizations as well.

Principles of Operation

The ^3He nucleus posseses both intrinsic spin angular momentum and a magnetic dipole moment which is directed antiparallel to the spin axis. However, a sample of ^3He will not generally possess any net angular momentum or magnetization due to the random orientation of the individual spins. The process of optical pumping[7] is employed to orient the individual spins along a preferred direction and thus achieve a net sample angular momentum.

When such a polarized sample is placed in a uniform magnetic field, $\boldsymbol{B}$, two processes ensue. The first is a relaxation of the sample back to its unpolarized equilibrium condition (assuming the optical pumping process is terminated). This effect causes the sample angular momentum to diminish with some characteristic decay time. For a cryogenic ^3He gyro this characteristic decay time can be made on the order of days. Thus, this effect is ignored in this analysis. The second process is due to the applied field interacting with the sample magnetization to produce a torque, $\boldsymbol{M}\times\boldsymbol{B}$. This torque equates to the time rate of change of the sample angular mometum in an inertial frame.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{H}=\boldsymbol{M}\times\boldsymbol{B} \tag{1}$$

However, $\boldsymbol{H}$ and $\boldsymbol{M}$ are antiparallel and, in fact, are related by the gyromagnetic ratio γ, which is a constant for the species.

$$\boldsymbol{M}=\gamma\boldsymbol{H} \tag{2}$$

Thus the equation of motion can be written solely in terms of $\boldsymbol{M}$.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{M}=\boldsymbol{M}\times\gamma\boldsymbol{B} \tag{3}$$

This equation holds for an inertial frame; but $\boldsymbol{B}$ is tied to the gyro. Then if the rotation rate of the gyro, with respect to the inertial frame, is ω, the equation of motion in the gyro frame is

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{M}=\boldsymbol{M}\times\gamma\boldsymbol{B}-\omega\times\boldsymbol{M} \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{M}=\boldsymbol{M}\times(\gamma\boldsymbol{B}+\omega) \tag{5}$$

Thus, the effects of a gyro rotation are to shift both the precession frequency with respect to the gyro frame and the axis of precession. The frequency shift can be observed with a sensitive magnetometer oriented with its input axis orthogonal to $\boldsymbol{B}$. This frequency shift then serves as a measure of rotation rate. The greatest sensitivity occurs when $\boldsymbol{B}$ and ω are parallel and thus the nuclear gyro is considered a single-degree-of-freedom gyro. It is clear, however, that cross-axis rates will induce frequency shifts also.[8,9] If $\gamma\boldsymbol{B}$ is made much large than ω, the cross-axis effects can be made arbitrarily small. There are, however, several good reasons to keep $\boldsymbol{B}$ as small as possible.

Research is ongoing to develop a method of utilizing three nuclear gyros with their sensitive axes orthogonal in a manner that exploits this cross-axis sensitivity to get an optimal estimate of ω. The intent is to be able to operate at low $\boldsymbol{B}$ (hence high cross-axis sensitivities). Because the greatest difficulty occurs for rotation rates very near $\gamma\boldsymbol{B}$ (the Larmor rate) much effort has been concentrated there.[9]

Effect of Oscillatory Cross-Axis Rate Inputs

One problem that has arisen involves oscillatory rates. In particular, if the nucler gyro experiences an oscillatory rate

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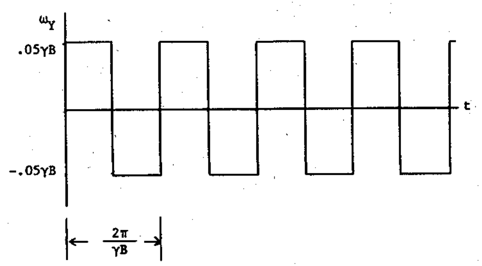

Fig. 1 Simulated oscillatory Y-axis rate.

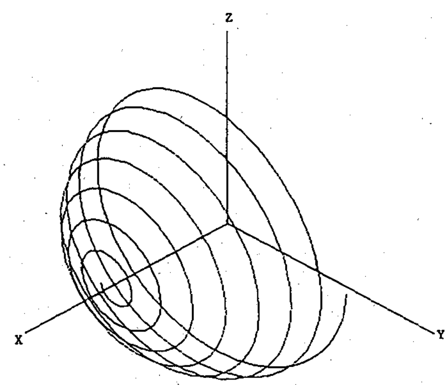

Fig. 2 Effect on X-axis gyro of oscillatory rate about Y-axis.

about a cross axis and at a frequency corresponding to the Larmor frequency, a periodic loss of magnetometer output results. This was demonstrated with a digital simulation of the ^3He gyro dynamics. The particular case demonstrated was an X-axis gyro in response to an oscillating rate about the Y axis. The cross-axis rate input (Fig. 1) is described by

$$\omega_Y = 0.05\gamma B \mathrm{sgn}[\cos\gamma B t] \tag{6}$$

The resulting path of the magnetization vector, as seen in the gyro frame, is shown to wind its way up the gyro X axis (Fig. 2). The magnitude of M is unchanged since relaxation effects are ignored, but if the oscillatory input persists, M winds back down into the $X=0$ plane and then continues its way down the $-X$ axis and back and forth. Since the magnetometer senses the component of M orthogonal to B the effect is to alternately diminish and restore the magnetometer signal. Overall then, the signal-to-noise ratio of the instrument is decreased. In this demonstration time has been scaled in terms of γB. For example, if γB is shosen to be 2π rad/s, then ω_Y has a peak rate of 0.31 rad/s and the total time shown (Fig. 2) is about 7.5 s. In other words, if the peak input rate is 5% of the Larmor rate, it will take about 7.5 Larmor periods for M to wind its way out of the $X=0$ plane.

The situation is most easily explained in a rotating frame. For example, approximate ω_Y as

$$\omega_Y = 0.05\gamma B \cos\gamma B t \tag{7}$$

Then represent ω_Y as two counter-rotating vectors in the $X=0$ plane, each of magnitude 0.25 γB, starting in the Y-axis direction and rotating at γB rad/s. Then, in the rotating frame, the vector traveling with the rotating frame is stationary, while the other appears to rotate at twice the Larmor rate. The effects of the latter average to zero, but the former causes M to precess about the rotating frame Y axis. In the gyro frame both the Y axis motion and the Larmor precession about the X axis cause the spiral trajectory shown. This situation is a worst-case situation. For oscillatory rates much higher or lower than γB, the effect averages to zero.

Conclusions

Cross-axis rates, oscillating at the Larmor frequency, will cause a degradation in the signal-to-noise ratio of nuclear magnetic resonance gyros. This has been demonstrated for a single-species, unpumped device but should cause related problems in the dual-species, continuously pumped devices as well. Care must be taken with such instruments to isolate them from vibrations at the Larmor frequency.

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An Exact Expression for Computing the Degree of Controllability

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Introduction

THE concept of degree of controllability was recently introduced to study control systems associated with large flexible spacecraft.[1-5] However, a formula for the exact value of the degree of controllability was not determined; instead various techniques for computing an estimate were developed in Refs. 1-5. Here we give an exact formula which can be used to compute the degree of controllability.

In the next section, we review the concepts and results associated with the idea of degree of controllabiltiy and then derive the above mentioned result. In a subsequent section, this result is used to compute the degree of controllability for several of the examples in Refs. 1-5.

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